Modeling Curvature Damage Surface for Damage Detection in Cantilever Beam

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***Abstract*— A damage detection method is presented for the identification and quantification of damage. The proposed method uses finite element method to extract modal parameters of cracked and intact cantilever beam. The damage is simulated by fracture mechanics concept by introducing cracked elements at different locations with predetermined magnitude of depth. The curvature response function, function of crack location and size, are approximated by means of polynomial surface fitting. The numerical data obtained is meshed using B-spline. The algorithm based on curvature, Wavelet Transform and surface fitting technique is proposed for damage detection. Cubic fit and quadratic fit are used for function generation and three dimensional plots. Number of numerical examples is presented to demonstrate the accuracy of the proposed methodology.**

***Index Terms*— Crack beam modeling, Response Curvature damage surface, Wavelet Transform, Response Curvature error damage surface.**

I. Introduction

The Vibration Based Inspection Methods (VBI) are popular because of convenience of measurement and collection of modal parameters. The global nature of the parameters (such as natural frequencies) implies that tests can be conducted at virtually arbitrary points. A prior knowledge of the damage location is not necessary. VBI methods also detect Hidden damage which is normally difficult to access. This capability draws attention from mathematicians and engineers during last decade. The crack beam modeling is important for numerical simulation. Modeling cracked beam element using the linear facture mechanics concepts [1] [2] will help in modeling hairline crack. An analytical as well as experimental method is used for crack detection in cantilever beams [3]. To locate the crack, contours of the normalized frequency in terms of the normalized crack depth and location are plotted. The intersection of contours with the constant modal natural frequency planes is used to relate the crack location and depth. The transverse crack is also modeled in a beam as rotational spring having stiffness of negligible mass [4]. For the computational convenience lumped mass matrix instead of consistent mass matrix is considered. The applications of Wavelet Transform to detect crack-like damage in structures are demonstrated. [5] [6] [7] This paper presents a method for damage detection based on the Response Curvature Damage Surface. The surface is generated from the curvature of mode shapes for first five modes. Additional data points are obtained by fitting B-spline curve. Curvature damage response surface function, curvature error damage response surface function and curvature resultant damage response functions are obtained from polynomial fitting of numerical data. The wavelet transform is used to analyze curvature mode shapes which locate the crack. The algorithm addresses the determination of crack depth using the damage response surface fitting polynomial.

II. Crack beam element modeling

According to the principle of Saint-Venant's, the stress field is affected only in the region adjacent to the crack. It is difficult to find an appropriate shape function to approximate the kinetic energy and elastic potential energy because of the discontinuity of deformation in crack element. The calculation of the additional stress energy due to crack is computed using the concept in fracture mechanics .The flexibility coefficient expressed by a stress intensity factor is derived by applying the Castigliano's theorem in the linear-elastic range. A crack model beam can be divided into elements and the behavior of the elements located to the right of the cracked element may be regarded as external forces applied to the cracked element, while the behavior of elements situated to its left as constraints . Thus, the flexibility matrix of a cracked element with constraints is calculated. Neglecting shear action, the strain energy of an undamaged element is

$$W^{(0)} = \frac{1}{2EI}(M^2L + MPL^2 + P^2L^3/3) \qquad (1)$$

Where E is the elastic modulus, P and M are the shear and bending internal forces at the right node, I the moment of inertia of the undamaged element and L the length of the finite element. For a rectangular beam having width b and thickness h additional strain energy due to the crack can be

$$W^{(1)} = b\int_0^a \left[\frac{(K_I^2 + K_{II}^2)}{E_p} + \frac{(1+\nu)K_{III}^2)}{E} \right] da \qquad (2)$$

Where K_I,K_{II} & K_{III} are stress intensity factors for opening type, sliding type and tearing type cracks, respectively. $Ep=E$ for plane stress, $Ep=E/(1-\nu^2)$ for plane strain, and a is the crack depth. Taking into account only bending, equation (2) becomes

$$W^{(1)} = b\int_0^a \left\{ \left[\left(K_{IM} + K_{IP}\right)^2 + K_{IIP}^2 \right] / E_P \right\} da \qquad (3)$$

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Where K_{IM}, K_{IP}, K_{IIP} *are* stress intensity factors for opening-type and sliding-type cracks due to *M* and *P* respectively

$$K_{IM} = \left(6M / bh^2\right)\sqrt{\pi a}F_I(s)$$
$$K_{IP} = \left(3PL / bh^2\right)\sqrt{\pi a}F_I(s) \qquad (4)$$
$$K_{IIP} = \left(P / bh\right)\sqrt{\pi a}F_{II}(s)$$

$F_I(s)$ and $F_{II}(s)$ are function of the ratio s between the crack depth and the height of the element ($s=a/h$), defined as

$$F_I(s) = \sqrt{(2/\pi s)tg(\pi s/2)}\,\frac{0.923+0.199\left[1-\sin(\pi s/2)\right]^4}{\cos(\pi s/2)}$$
$$F_{II}(s) = \left(3s-2s^2\right)\frac{1.122-0.561s+0.085s^2+0.18s^3}{\sqrt{1-s}} \qquad (5)$$

The flexibility coefficient of the intact element can be derived as

$$C_{ij}^{0} = \frac{\partial^2 W^{(0)}}{\partial P_i \partial P_j} \qquad (6)$$
$$P_1 = P, \quad P_2 = M, \quad i, j = 1, 2$$

The additional flexibility coefficients are

$$C_{ij}^{1} = \frac{\partial^2 W^{(1)}}{\partial P_i \partial P_j} \qquad (7)$$
$$P_1 = P, \quad P_2 = M, \quad i, j = 1, 2$$

The flexibility matrix of the intact element is

$$\left[C^{(0)}\right] = \frac{1}{EI}\begin{pmatrix} \frac{L^3}{3} & \frac{L^2}{2} \\ \frac{L^2}{2} & L \end{pmatrix} \qquad (8)$$

Similarly the coefficients $C_{ij}^{(1)}$ can be expressed in matrix form as

$$\left[C^{(0)}\right] = \frac{b\pi a^2}{E_p}\begin{pmatrix} 9\beta_1^2 L^2 + \beta_2^2 & 18\beta_1^2 L \\ 18\beta_1^2 L & 36\beta_1^2 \end{pmatrix} \qquad (9)$$

$$Where\ \beta_1 = F_I(s)/bh^2 and\ \beta_2 = F_{II}(s)/bh$$

The total flexibility coefficients for the element with crack are

$$C_{ij} = C_{ij}^{(0)} + C_{ij}^{(1)} \qquad (10)$$

The total flexibility matrix for the element with crack can be expressed as

$$\left[C\right] = \left[C^{(0)}\right] + \left[C^{(1)}\right] \qquad (11)$$

From the equilibrium conditions the following relationship holds

$$\{P_i \quad M_i \quad P_{i+1} \quad M_{i+1}\}^T = [T]\{P_{i+1} \quad M_{i+1}\}^T \qquad (12)$$

Where

$$[T] = \begin{bmatrix} -1 & -L & 1 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix}^T \qquad (13)$$

The stiffness matrix of the intact element can be written as

$$\left[K_u\right] = [T]\left[C^{(0)}\right]^{-1}[T]^T \qquad (14)$$

The stiffness matrix of the cracked element can be written as

$$\left[K_C\right] = [T][C]^{-1}[T]^T \qquad (15)$$

The Euler-Bernoulli's beam element is used in the formulation. It is assumed that the crack present in the element affect only stiffness but not the mass. The cracked beam element is introduced with concept of linear fracture mechanics. The free vibration analysis of the beam is performed using MATLAB code. The consistent mass matrix and stiffness matrix for beam element is used. to analyze the intact beam. The algorithm proposed in the paper follows.

III. Algorithm For Damge Detection

The proposed algorithm for damage detection is

- Forward problem

1. Model the structure for solving eigenvalue problem in finite element method.
2. Use model updating to match the responses from finite element method and actual response.
3. Obtain Normalized mode shapes and frequencies from the eigen solution. Curvature of mode shapes is obtained and used for damage location.

Damage response surface function, Error damage response surface function and Resultant damage response function are established as a function of location of crack from fixed end (Lcr) and crack depth ratio (a/h). Few typical fit can be mathematically written as

- Quadratic fit surface polynomial

$Curd = A + BL_{CR} + CY + DL_{CR}^2 + EY^2 + FL_{CR}Y$
&
$Error = A_1 + B_1L_{CR} + C_1Y + D_1L_{CR}^2 + E_1Y^2 + F_1L_{CR}Y$

- Cubic fit surface polynomial

$Curd = A + BL_{CR} + CY + DL_{CR}^2 + EY^2 + FL_{CR}Y + GL_{CR}^3 + HY^3 + IL_{CR}^2Y + {}^{J}L_{CR}Y^2$
&
$Error\ Curd = A_1 + B_1L_{CR} + C_1Y + D_1L_{CR}^2 E_1Y^2 + F_1L_{CR}Y + G_1L_{CR}^3 + H_1Y^3 + I_1L_{CR}^2Y + {}^{J}{}_1L_{CR}Y^2$

Where Curd = curvature. Y= (a/h) = Crack depth ratio & L_{CR} = Crack location from fixed end
The resultant curvature and error curvature are calculated as follows

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Resultant Curd = Curd – error.
Error Curd = Curd new – Curd

➢ Inverse problem

To simulate real life situation Noise added in mode shapes. Crack location is located by wavelet transform or curvature of mode shape. Crack depth ratio (a/h) is expressed as a function of location of crack from fixed end (Lcr) and Curvature i.e.

$$\frac{a}{h} = F_i(\mathrm{Lcr}, \mathrm{Curvature}) \quad \text{where } i = 1,2,3 \ldots . N$$

Construction and plotting of Damage response surface function, Error damage response surface function and Resultant damage response function. .Percentage error is established from the numerical simulated data.

IV. NUMERICAL SIMULATION

To implement the proposed algorithm, a cantilever beam is considered with following geometrical parameters. The beam is 0.3 m long with cross section 0.0225 m x0.013 m respectively.. The material properties taken are: modulus of elasticity 175 GPa, mass density 7.8 kg/m3, and Poisson ratio 0.3. The crack depth '*a*' and crack location L_{cr} are varied during the analysis. Based on the proposed algorithm a code was generated. Numerical simulation obtained by finite element by introducing crack at a location with variation in crack depth. The crack locations are varied from 0 to 300 in a step of 50 while the crack depth ratio varied from intact to 0.6. The code prompts user to solve the problem with or without Guyan reduction technique. The eigenvalue problem is solved to obtain modal parameters.

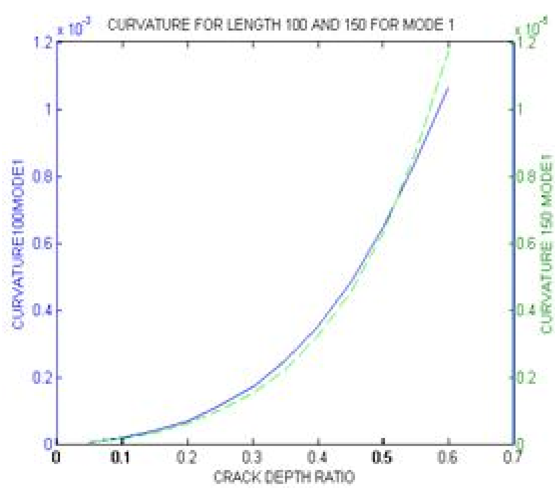

Figure 1. Curvature with varying Crack depth (for length 100 and 150 (mode 1))

The curvature mode shapes are evaluated. The variation of curvature increases as the crack depth increases. Additional data is generated by meshing the numerical data with spline function.

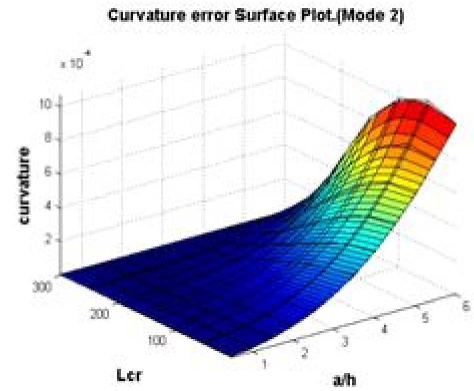

Figure 2. Spline fitted Curvature Surface plot.

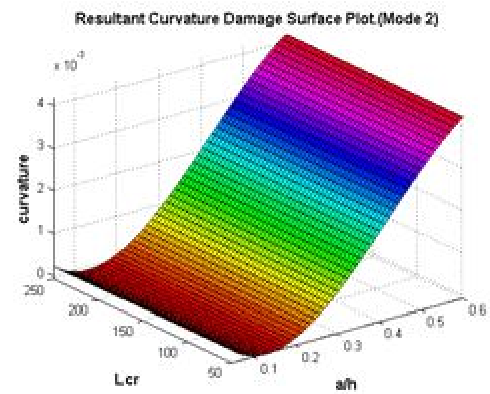

Figure 3. Damage curvature response surface plot

The damage surface for the curvature is generated using response surface method. MATLAB code is used for obtaining coefficients of quadratic and cubic polynomial fit. Such polynomial fit is obtained for first five mode shape. Equation below is a quadratic polynomial fit for mode 3.

$$\mathrm{Curd}=1.23e\text{-}03+5.51e\text{-}05{*}L_{CR}+7.78e\text{-}03{*}Y$$

$$+2.02e\text{-}07{*}L_{CR}^{2}+4.27e\text{-}2{*}Y^{2}\ +1.47e\text{-}05{*}L_{CR}Y$$

Construction and plotting of Damage response surface function, Error damage response surface function and Resultant damage response function as a function of location of crack from fixed end (Lcr) and crack depth ratio (a/h) from numerical data. Similarly, Construction and plotting of Damage response surface function, Error damage response surface function and Resultant damage response function. Crack depth ratio (a/h) is expressed as a function of location of crack from fixed end (Lcr) and Curvature i.e.

$$\frac{a}{h} = F_i(\mathrm{Lcr}, \mathrm{Curvature}) \quad \text{where } i = 1,2,3 \ldots . N$$

The inverse problem is used for damage detection. To simulate real life situation Noise added in Normalize mode shapes. The wavelet transform (figure 4) is used for locating crack the location of crack is verified by plotting the curvature mode shape. Even multiple cracks are also located by Wavelet transform.

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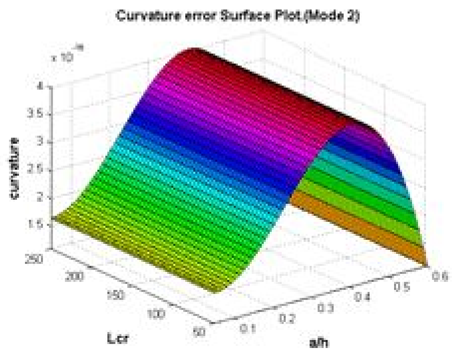

Figure 4. Error damage curvature response Surface plot

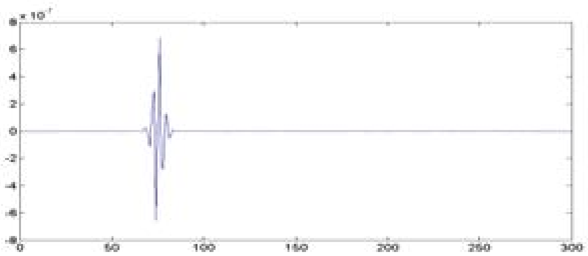

Figure 5. Crack detection at element number 75 (with crack depth 0.05)

Resultant damage response polynomial function in terms of the Crack depth ratio (a/h) is expressed as a function of location of crack from fixed end (Lcr) and Curvature from numerical data. This function is used to establish Crack depth ratio.

TABLE 1
PERCENTAGE ERROR IN PREDICTING CRACK DEPTH RATIO

Crack Depth Ratio / Fit	Length 200 Quadratic	Length 200 Quadratic	Length 250 Quadratic	Length 250 Cubic
Mode	1	2	3	2
0.05	-14.38	-127.70	-85.64	-53.89
0.10	11.99	-29.88	-12.56	-14.19
0.15	10.88	-3.38	4.83	1.89
0.20	4.41	5.09	8.68	7.81
0.25	-2.76	5.92	7.38	7.80
0.30	-8.32	2.58	4.09	3.69
0.35	-10.39	-3.14	0.89	-3.29
0.40	-7.84	-9.65	-0.52	-11.40
0.45	-1.71	-14.92	0.79	-17.45
0.50	3.27	-16.24	3.59	-16.29
0.55	-0.13	-10.59	3.35	-2.05

Table 1 shows Percentage error in predicting Crack depth ratio for few cases.

The percentage error indicates the lower percentage error for most of the crack depth ratio except for the lower crack depth ratio. The percentage error is reduced in case of cubic fit but error increases at lower crack depth.

IV. Conclusions

The present study addresses on the identification of the presence, location and size of a crack in a cantilever beam structure by extracting mode shapes and frequencies from numerical simulation. The eigenvalue problem is solved to obtain the modal parameters. The algorithm presented based on curvature mode shape which is used for damage detection. The new concept of error surface plot is introduced for efficient and accurate prediction of crack location and size. The wavelet transform shows the sharp peaks at the crack location and depth is established by response surface fitting function. The error in the prediction of crack depth ratio is higher for lower crack depth ratio. The technique is applicable to any linear structure that can be accurately modeled using finite element method.

REFERENCES

[1] R. Ruotolo, C. Surace, P. Crespo, D. Strorer. Harmonic analysis of the vibrations of a cantilevered beam with a closing crack. Computers & structures Vol.61 pp 1057-1074, 1996..

[2] G.M. Owolabi, A. S.J. Swamidas and R. Seshadri, Crack detection in beams using changes in frequencies and amplitudes of frequency response functions" Journal of sound and vibration Vol. 265, 1-22. 2003.

[3] H. Nahvi, M. Jabbari. Crack detection in beams using experimental modal data and finite element model. International Journal of Mechanical Science Vol 47, pp 1477-1497, 2005.

[4] S.Chinchalkar. Determination of crack location in beams using natural frequencies. Journal of Sound and vibration 247(3), pp 417-429, 2001.

[5] A.V.Ovanesova, L.E.Suarez. Applications of Wavelet Transforms to damage detection in frame structure. Engineering Structure, Vol 26, pp 39-49, 2004.

[6] Hansang Kim, Hani Melhem. Damage detection of structures by Wavelet analysis. Engineering Structure, Vol 26, pp 347-362, 2004.

[7] Hansang Kim, Hani Melhem. Damage detection of structures by Wavelet analysis. Engineering Structure, Vol 26, pp 347-362, 2004.

[8] E. Douka, S. Loutridis, A. Trochidis. Crack identification in beams using Wavelet analysis" Journal of Sound and Vibration vol 40, pp 3557–3569, 2003.

[9] A.A.,Lonkar Venkataswami Ramala and R.K. Srivastava. "Multiple Crack detection in Cantilever beam using Wavelet Transform." Proceedings of International Conference Computer Aided Engineering. (CAE2007), Department of mechanical engineering, I.I.T. Madras, India. pp 201 to 209.

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